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SECOND ORDER ITÔ PROCESSES

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1. Introduction. A first order stochastic differential equation is any equation which can be expressed symbolically in the form

$$dy(t) = m[t, y(t)]dt + \sigma[t, y(t)]dz(t); \tag{1. 1}$$

m and σ are called the drift and diffusion coefficients and $z(\cdot)$ is usually a Brownian motion process. If $m[t, x] \equiv m_0 x$ and $\sigma[t, x] \equiv \sigma_0$, where m_0 and σ_0 are constants, then this equation is called the Langevin equation, and its importance has been recognized for some time in many problems of physics and engineering. The rigorous interpretation and the development of the corresponding theory of the y-process, with the Itô-Doob approach to stochastic integrals, comprises part of diffusion theory (i.e. the theory of Markov processes with continuous sample paths) and is treated in detail in the recent books of Dynkin [5] and Skorokhod [18].

The following related problem has received little attention thus far. It concerns the simple harmonic oscillator driven by a Brownian disturbance (i.e. "white noise"), given by the symbolic stochastic equation

$$dy'(t) + 2\alpha y'(t)dt + \beta^2 y(t)dt = dz(t),$$

where y' denotes the sample derivative of the y-process describing the position of the particle, the z-process again being Brownian motion. This type of equation leads naturally to non-linear extensions of the form

$$dy'(t) = m[t, y(t), y'(t)]\, dt + \sigma[t, y(t), y'(t)]dz(t). \tag{1. 2}$$

In this paper we shall study the stochastic processes satisfying equation (1. 2). The solution process, i.e. the y-process, will be called a *second order Itô process* following the terminology introduced by Borchers [2].

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