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ABSOLUTE CONTINUITY OF MARKOV PROCESSES AND GENERATORS

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Introduction.

Let $(x_t, \zeta, \mathfrak{B}_t, P_x)$ be a (standard) Markov process with state space S defined on the abstract space Ω. Here, x_t is the sample path, ζ is the terminal time and $\mathfrak{B}_t$ is the smallest σ-field of Ω in which x_s, $s \leqq t$ are measurable. Let P'_x, $x \in S$ be another family of Markovian measures defined on $(\mathfrak{B}_t, \Omega)$. It is a known fact that $(\mathfrak{B}_t[t<\zeta], P'_x)$ is absolutely continuous with respect to $(\mathfrak{B}_t[t<\zeta], P_x)$ for any $t>0$ and $x \in S$, if and only if there exists a positive right continuous multiplicative functional (MF) M_t with $P_x(M_t) \leqq 1$, $x \in S$, $t \geqq 0$, such that it is the Radon-Nikodym derivative of $(\mathfrak{B}_t[t<\zeta], P'_x)$ with respect to $(\mathfrak{B}_t[t<\zeta], P_x)$, where $\mathfrak{B}_t[t<\zeta]$ is the σ-field in $[t<\zeta]$ formed by all $B \cap [t<\zeta]$, $B \in \mathfrak{B}_t$. Then there arises naturally the following problem; How can we characterize the class of all the Markov process which is absolutely continuous with respect to a given Markov process or, equivalently, the class of all the Markov process which is transformed through MF of a given Markov process? In particular can we characterize this class in terms of the generator of Markov process?

In case of Brownian motion, this problem is solved through the works of Maruyama [6], Motoo [8], Dynkin [1] and Wentzell [15]. It is roughly the following; the conservative Markov process which is absolutely continuous with respect to Brownian motion has the generator expressed as $\frac{1}{2}\Delta + \sum f_i \frac{\partial}{\partial x_i}$: Hence the transformation by MF is so-called that of "drift". On the other hand the same problem has been solved in case of Markov chain by Kunita-Watanabe [4]; two (minimal) Markov chains x_t and x'_t with the same state space S are mutually absolutely continuous if and only if $q_{x,y}=0$ implies $q'_{x,y}=0$ and vice versa, where $q_{x,y}=\lim_{t \downarrow 0} \frac{P_t(x,y)}{t}$ and $P_t(x,y)$ is the transition function of x_t ($q'_{x,y}$ is defined similarly from x'_t).

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