O.E. Villamayor and D. Zelinsky
Nagoya Math. J.
Vol. 35 (1969), 83–98

GALOIS THEORY WITH INFINITELY MANY IDEMPOTENTS[1)]

O.E. VILLAMAYOR AND D. ZELINSKY

1. Introduction.

In 1942 Artin proved the linear independence, over a field S, of distinct automorphism of S; in other words if G is a finite group of automorphisms of S and R is the fixed field, then $\mathrm{Hom}_R(S,S)$ is a free S-module with G as basis. Since then, this last condition ("S is G-Galois") or its equivalents have been used as a postulate in all the Galois theories of rings that are not fields, for example by Dieudonné, Jacobson, Azumaya and Nakayama for noncommutative rings and then in [*AG*, Appendix] and [*CHR*] for commutative rings. When S has no idempotents but 0 and 1, [*CHR*] proves that the ordinary fundamental theorem of Galois theory holds with no real change from the classical, field case.

If the rings have finitely many idempotents, the G-Galois condition prevents the "Galois group" G from being the full automorphism group, but [*CHR*] provides a Galois theory pairing all subgroups of G with certain separable subalgebras. In a sense this is a study of the group G as a transformation group on a commutative ring S. In [*VZ*] we presented a different Galois theory, oriented toward the rings rather than the groups, pairing all separable R-subalgebras of S with some subgroups of the full automorphism group of S over R. The present paper contains the same Galois theory, with no hypotheses at all on idempotents. The technique uses Pierce's representation [*P*] of the ground ring R as the global cross sections of a sheaf of rings that have no nontrivial idempotents, so that at each point x of the base space we have a ring extension of R_x to which [*VZ*] applies.

In order to carry out this program, the G-Galois condition is too restrictive. Our hypothesis, besides the natural finite generation, projectivity,

Received July 13, 1968

1) This research was made possible by N.S.F. Grant GP–1649, Travel Grant GP–8231, and the Conference on Rings and Modules at Oberwolfach, March 1968.