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ON A CROSSED PRODUCT OF A DIVISION RING

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1. Let R and C be a ring and its center, and G an automorphism group of R of order n. By a factor set $\{c_{\sigma,\tau}\}$, we mean a system of regular elements $c_{\sigma,\tau}$ $(\sigma,\tau \in G)$ in C such that

$$(1) \qquad c_{\sigma,\tau\rho}c_{\tau,\rho} = c_{\sigma\tau,\rho}c_{\sigma,\tau}^{\rho}.$$

A crossed product $W = W(R, G, \{c_{\sigma,\tau}\})$ is a ring containing R such that $W = \sum_{\sigma\in G} u_\sigma R$ (direct) with regular elements u_σ and $au_\sigma = u_\sigma a^\sigma$ for a in R and $u_\sigma u_\tau = u_{\sigma\tau}c_{\sigma,\tau}$. As usual, we identify $W(R, G, \{c_{\sigma,\tau}\})$ and $W(R,G, \{c'_{\sigma,\tau}\})$ when $c_{\sigma,\tau}$ and $c'_{\sigma,\tau}$ are cohomologous (in C). When $c_{\sigma,\tau} = 1$, the crossed product is called splitting. In this note, we shall deal with a division ring D as R, and when $S = \{a \in D \mid a^\sigma = a$ for all σ in $G\}$, we suppose $[D:S]=n$. In this case, D/S is called a strictly Galois extension with a Galois group G([3],[4]). The purpose of this note is to discuss a splitting property of W by extending the base ring S as well as D, which is an analogy of the classical result of commutative case. We shall show that there exist a division ring D' such that $S \subseteq D' \subseteq D$ and a kind of (non-commutative) Kronecker product $D^* = D \otimes D'$ over S such that $W(D^*, G, \{c_{\sigma,\tau}\})$ becomes splitting. The construction of the Kronecker product seems very interesting to the author and an example will be given in the last section.

2. Let D be a division ring and $x_1, \cdots, x_m$ m indeterminates. A polynomial ring $D[x_1, \cdots, x_m]$ is defined in a natural way, supposing commutativity of multiplication between elements of D and x_i and between x_i and x_j. The quotient division ring of $D[x_1, \cdots, x_m]$ is called the rational function division ring, whose existence is almost clear when we imbed $D[x_1, \cdots, x_m]$ into the formal power series division ring $D\{x_1, \cdots, x_m\} = D\{x_m\}\{x_{m-1}\}\cdots\{x_1\}$ of $x_1, \cdots, x_m$ over D and take the

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