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CONTRACTION GROUPS AND EQUIVALENT NORMS*

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Using the notation in [1], the Lumer-Phillips theorem (3. 1 of [2]) is refined to single parameter groups in real Banach space and real Hilbert space. The theory can be extended to complex spaces.

DEFINITION 1.

Let X be a B-space with norm $\|\cdot\|_1$ and let $[\cdot,\cdot]_1$ be a corresponding semi-scalar product on X. Then the semi-scalar product $[\cdot,\cdot]$ is said to be equivalent to $[\cdot,\cdot]_1$ on X iff $\|\cdot\|_1$ and $\|\cdot\|$ are equivalent norms on X.

THEOREM 1.

Let A be a linear operator with $D(A)$ and $R(A)$ both contained in a B-space $(X, \|\cdot\|_1)$ such that $D(A)$ is dense in X. Then A generates a group $\{T_t; -\infty < t < \infty\}$ in X such that $\{T_t;\ t > 0\}$ is a negative contractive semi-group with respect to an equivalent norm $\|\cdot\|$ iff

$$-\delta\|x\|^2 < [Ax, x] < -\gamma\|x\|^2 \qquad (x \in D(A)) \tag{1}$$

where $\infty > \delta > \gamma > 0$ and $[\cdot,\cdot]$ is an equivalent scalar product consistent with $\|\cdot\|$, and

$$R(I(1-\gamma) - A) = X \qquad R(I(1+\delta) + A) = X. \tag{2}$$

Proof.

The sufficiency of conditions (1) and (2) follows immediately from the results in Yosida [1], pp. 250–254.

Conversely suppose that A generates a group such that $\|T_t\| < e^{-\beta t}$ $(t \geqq 0)$ where $\beta > 0$. It is known that for a group $\|T_t^{-1}\| < Me^{\alpha t}$, where

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