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ON UNIFORM APPROXIMATION BY RATIONAL FUNCTIONS WITH AN APPLICATION TO CHORDAL CLUSTER SETS*

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For a closed and bounded set E in the complex plane, let $A(E)$ denote the collection of all functions continuous on E and analytic on E°, its interior; let $R(E)$ denote the collection of all functions which are uniform limits on E of rational functions with poles outside E. Then let $\mathscr{A}$ denote the collection of all closed, bounded sets for which $A(E) = R(E)$. The purpose of this paper is to formulate a condition on a set, which is essentially of a geometric nature, in order that the set belong to $\mathscr{A}$. Then using approximation techniques, we shall construct a meromorphic function having a certain boundary behavior on a perfect set; this answers a question raised in [1].

Uniform Approximation

For any subset H of the complex plane, let $C(H)$ denote the set of all functions each of which is continuous on the whole plane, analytic outside some closed subset of H, bounded in modulus by the constant one, and equal to zero at infinity. Let

$$\alpha(H) = \sup_{f \in C(H)} \lim_{z \to \infty} |zf(z)|.$$

Then $\alpha(H)$ is called the analytic C-capacity of H.

The result we obtain does not depend on the rather complicated definition of the analytic C-capacity of a set, but depends instead only on the formal relationship appearing in the following theorem of A.G. Vituškin [6, Theorem 2].

THEOREM A. *Let E be a closed and bounded set. Then $E \in \mathscr{A}$ if and only if for every open set G, the equality $\alpha(G - E) = \alpha(G - E^\circ)$ is satisfied.*

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