K. Amano
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A NOTE ON GALOIS COHOMOLOGY GROUPS OF ALGEBRAIC TORI

KAZUO AMANO

§1. Introduction

Let k be a complete field of characteristic 0 whose topology is defined by a discrete valuation and let T be an algebraic torus of dimension d defined over k. As is well known, T has a splitting field K which is a finite Galois extension of k with Galois group $\mathfrak{G}$. For a ring R, denote by T_R the subgroup of R-rational points of T. Then T_K and $T_{\mathfrak{o}_K}$, $\mathfrak{o}_K$ being a valuation ring of K, become $\mathfrak{G}$-modules in the usual manner.

In the present paper, we shall show some properties of $\mathfrak{G}$-modules T_K and $T_{\mathfrak{o}_K}$. Namely, in Section 2, we shall obtain Theorem 1 as an analogy to the results as is well known in the local fields. In Section 3, we shall consider the Galois cohomology groups of T_K and $T_{\mathfrak{o}_K}$ as $\mathfrak{G}$-modules [Theorem 2]. Analogous results in the case of number fields were obtained in [11] and [15]. In Section 4, we shall obtain the explicit structure of the Galois cohomology groups of $T_{\mathfrak{o}_K}$ for the totally ramified extension of prime degree.

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§2. Unramified extension

In this section, we suppose that the splitting field K is always an unramified extension of k. We denote by $\mathfrak{u}_K$ (resp. $\mathfrak{u}_k$) the group of units of K (resp. k). For a unique prime divisor $\mathfrak{P}$ (resp. $\mathfrak{p}$) of K, we set for the integer $r \geqq 0$

$$\mathfrak{u}_K^{(r)} = \{\alpha \in \mathfrak{u}_K,\ \alpha \equiv 1 \bmod.\ \mathfrak{P}^r\},\quad \mathfrak{u}_K^{(0)} = \mathfrak{u}_K,$$
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and define $T_{\mathfrak{o}_K}^{(r)}$ by

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