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ON THE CONTINUITY OF STATIONARY GAUSSIAN PROCESSES

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1. Introduction

Let us consider a stochastically continuous, separable and measurable stationary Gaussian process[1] $\boldsymbol{X} = \{X(t),\ -\infty < t < \infty\}$ with mean zero and with the covariance function $\rho(t) = EX(t+s)X(s)$. The conditions for continuity of paths have been studied by many authors from various viewpoints. For example, Dudley [3] studied from the viewpoint of ε-entropy and Kahane [5] showed the necessary and sufficient condition in some special case, using the rather neat method of Fourier series.

In this note we shall discuss the continuity of paths of $\boldsymbol{X}$, making use of the idea presented by Kahane. Our results are following: We express the covariance function ρ in the form

$$\rho(t) = \int_{-\infty}^{\infty} e^{it\lambda}\, dF(\lambda)$$

with a finite measure dF, symmetric with respect to origin.

Put $\qquad s_n = F(2^n, 2^{n+1}], \qquad n = 0, 1, 2, \cdots.$

THEOREM 1. *If* $E \sup_{t\in[0,1]} |X(t)| < \infty$, *then* $\sum_{n=0}^{\infty} \sqrt{s_n} < \infty$.

THEOREM 2. *Suppose that we can choose a decreasing sequence* $\{M_n\}$ *so that* $M_n \geq s_n$ *and* $\sum_{n=0}^{\infty} \sqrt{M_n} < \infty$. *Then* $E \sup_{t\in[0,1]} |X(t)| < \infty$.

THEOREM 3. *Suppose that* ρ *is convex on a small interval* $[0, \delta]$. *Then* $\sum_{n=0}^{\infty} \sqrt{s_n} < \infty$, *if* $\boldsymbol{X}$ *has continuous paths.*

By virtue of Theorem 2, we can easily see

COROLLARY. *Suppose that* ρ *is convex on a small interval* $[0, \delta]$ *and* s_n *is*

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[1] We mean a real valued process.