

A CATEGORICAL SETTING FOR DETERMINANTS AND TRACES

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The aim of this paper is to investigate some applications of a certain universal problem. The universal problem deals with categories \mathcal{C} which for every object A have some "structure" on the set $\mathcal{C}(A, A)$ of endomorphisms of A and calls for a universal solution, relative to this structure, that is associated with $\mathcal{C}(A, A)$ and centralizes the set of automorphisms of A . The commutative version of this universal problem asks for a universal solution, relative to the said structure, that abelianizes the canonical monoid structure of $\mathcal{C}(A, A)$.

In §1 the general case is discussed. A number of existence theorems, all versions of the Special Adjoint Functor Theorem (see [11]) are stated and various structure theorems concerning the universal solution are proved. §2 deals with presheaves and the corresponding universal problem. It is shown that the universal problem for presheaves may be solved pointwise and that, under fairly weak assumptions, the stalk functor commutes with the universal solution. It is also asserted that under appropriate hypotheses any recollatement of a sheaf leaves the universal solution unchanged. In §3 the trace for endomorphisms in an R -additive category is defined as a special instance of the universal problem of §1. Here, the previously mentioned structure is that of a left R -module. The existence of the trace (for the endomorphisms) of any object A is easily obtained. It turns out to be the canonical morphism from $\text{End } A$ to $H_0(\text{Aut } A, \text{End } A)$ with $\text{Aut } A$ operating on $\text{End } A$ by conjugation. Moreover it is shown that the trace of an endomorphism of a finite direct sum "is the sum of the diagonal entries" in the matrix description of that endomorphism. In §4 we restrict ourselves to the study of the trace for endomorphisms of unital R -modules, R being

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