ON THE FORMAL THETA FUNCTIONS OF
COUNTABLY MANY VARIABLES

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Introduction

In the previous paper [6] we have shown the examples of hyperelliptic Riemann surfaces of infinite genus such that the Riemann's theta functions associated with them are absolutely convergent.

In the present paper we shall study the formal properties of formal theta functions of countably many variables, analogeous to the case of finite variables [3], [4]: the canonical base and the addition formula etc. .

In §1, we shall define the formal theta functions of countably many variables with rational characteristics in the same way as [3], [4], and show the formal properties of these functions.

Section 2 is concerned with the special case: the infinite products of the elliptic theta functions with rational characteristics. We shall recall the sufficient conditions under which these theta functions are absolutely convergent and take Jacobi's expression of the elliptic theta functions as infinite products.[1] Using this expression we shall give the proofs of our results, which are analogeous to the case of finite variables [3], [4].

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Notations and conventions

Q^r: the coordinate vector space of dimension r over the rational number field Q,

Z^r: the subspace in Q^r consisting of all the integral vectors, i.e. vectors with integral coordinates,

Q^r/Z^r: the residue group of Q^r by Z^r,

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[1] This Jacobi's expression is a generalization of Jacobi's expression of so called Jacobi's theta function $\vartheta_3(z, q)$. See p. 464 and 469 in [7].