Samuel S.H.
Nagoya Math. J.
Vol. 33 (1968), 165–172

ON THE SUM OF HOMOLOGICAL DIMENSION AND CODIMENSION OF MODULES OVER A SEMI-LOCAL RING

SAMUEL S.H. YOUNG

Introduction.

Let M be a finitely generated module over a regular local ring R. It is well known that the sum of homological dimension and codimension of M is equal to the global dimension of R. For modules over an arbitrary ring, this is in general not true. The purpose of this paper is to investigate the properties of such sums in the semi-local case.

Throughout this paper, we shall use S to denote a semi-local ring, that is, a commutative noetherian ring with unity having only a finite number of maximal ideals. We shall also assume that S is of finite global dimension and that all S-modules are non-null, finitely generated and unitary. Known results are as a rule quoted without proof.

In Section 1, we collect some results concerning M-sequences and codimension. In Section 2, we generalize a proposition relative to local rings to the semi-local case. In Section 3, we define the pro-global and pro-total dimensions of S-modules and the total dimension of the ring S itself. Section 4 is concerned with the relations between the various dimensions. In Section 5, we show that the various dimensions remain unchanged on completion. The results are then applied in Section 6 to study the characterization of semi-local rings of total dimension 2.

Thanks are due to Professor Y. C. Wong for his encouragement and support and to Dr. K.T. Leung for first stimulating the author's interest in the theory of local rings and homology.

1. M-Sequences and Codimension.

DEFINITION 1. 1. Let M be a finitely generated module over a noetherian ring R. A sequence of elements $\{u_1, \cdots, u_t\}$ in an ideal $\mathfrak{n}$ of R is called an M-sequence in $\mathfrak{n}$ if u_{i+1} is not a zero divisor of $M/(u_1, \cdots, u_i)M$

Received January 22, 1968.