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ON SOME PROPERTIES OF NORMAL MEROMORPHIC FUNCTIONS IN THE UNIT DISC

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1. We denote by D the unit disc $\{z; |z| < 1\}$ and by $\mathscr{S}$ the totality of one to one conformal mappings $z' = s(z)$ of D onto itself. A meromorphic function $f(z)$ in D is normal if and only if the family $\{f(s(z))\}_{s(z)\in\mathscr{S}}$ is a normal family in D in the sense of Montel. We denote by $\mathfrak{N}$ the totality of the normal meromorphic functions in D. Moreover, Noshiro introduced in [5] the notion of the normal meromorphic functions of the first category: $f(z)$ is a normal meromorphic function of the first category if and only if $f(z)$ belongs to $\mathfrak{N}$ and any sequence $\{f_n(z)\}$ obtained from the family $\{f(s(z))\}_{s(z)\in\mathscr{S}}$ can not admit a constant as a limiting function. We denote by $\mathfrak{N}_1$ the totality of the normal meromorphic functions of the first category. For instance, Schwarzian triangle functions belong to $\mathfrak{N}_1$. In §1, we shall give a necessary condition (Th. 1) and a sufficient condition (Th. 2) for a function to belong to $\mathfrak{N}_1$. Further we shall give some properties of a function of $\mathfrak{N}_1$. In these proofs the Hurwitz theorem will play an essential role.

In 1957, Lehto and Virtanen ([4]) showed that even if $f(z)$ and $g(z)$ belong to $\mathfrak{N}$, $f(z)\pm g(z)$ and $f(z)g(z)$ do not necessarily belong to $\mathfrak{N}$. Later Lappan ([2], [3]) gave sufficient conditions for $f(z)\pm g(z)$ and $f(z)g(z)$ to belong to $\mathfrak{N}$. In §2, we shall give a more general sufficient condition for $f(z)g(z)$ to belong to $\mathfrak{N}$ than that of Lappan.

§1. Normal meromorphic functions of the first category

2. We consider the hyperbolic distance

$$d(z_1, z_2) = \frac{1}{2}\log\frac{|1-\bar{z}_1 z_2| + |z_1 - z_2|}{|1-\bar{z}_1 z_2| - |z_1 - z_2|}$$

for z_1 and z_2 in D, and the chordal distance

$$\chi(\alpha, \beta) = \frac{|\alpha - \beta|}{\sqrt{1+|\alpha|^2}\,\sqrt{1+|\beta|^2}}$$

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