H. Morikawa
Nagoya Math. J.
Vol. 33 (1968), 133-135

ON COMMUTATIVE COMPOSITIONS DETERMINED BY THEIR ORIGINS

HISASI MORIKAWA

1. Let K be the universal domain. Let G be a finite additive group of odd order $|G|$ and $X_a (a \in G)$ be indeterminates indexed by the elements in G. We mean by P_G the projective space $Proj_k(K[(X_a)_{a\in G}])$. Denote by δ_{-1} and $\tau_b(b \in G)$ the automorphisms of P_G of which duals δ^*_{-1} and τ^*_b are the ring-automorphisms of $Z[(X_a)_G]$ such that

$$\delta^*_{-1}(X_a) = X_{-a}, \quad \tau^*_b(X_a) = X_{b+a} \quad (a, b \in G).$$

For the sake of simplicity we denote briefly

$$x^{-1} = \delta^{-1}(x), \quad x(b) = \tau_b(x) \quad (x \in P_G, \quad b \in G).$$

DEFINITION 1. 1 Let $e = (e_a)_G$ be a point on P_G satisfying

(1) $\qquad e_{-a} = e_a \quad (a \in G).$

Then two points $x = (x_a)_G$ and $y = (y_a)_G$ are called to be composable with respect to e, if there exist two vectors $u = (u_a)_G$ and $v = (v_a)_G$ such that

$$(2) \qquad \operatorname{rank}\begin{pmatrix} (e_{-a+b}e_{a+b})_{G,G} & (y_{-a+d}\, y_{a+d})_{G,G} \\ {}^t(x_{-c+b}x_{c+b})_{G,G} & (u_{-c+d}v_{c+d})_{G,G} \end{pmatrix} = \operatorname{rank}(e_{-a+b}e_{a+b})_{G,G},$$

where $(e_{-a+b}e_{a+b})_{G,G}$, $(x_{-a+b}x_{a+b})_{G,G}$, $(y_{-a+b}y_{a+b})_{G,G}$ and $(u_{-a+b}v_{a+b})_{G,G}$ are $|G|+|G|$-matrices of which (a, b)-components are $e_{-a+b}e_{a+b}$, $x_{-a+b}x_{a+b}$, $y_{-a+b}y_{a+b}$ and $u_{-a+b}v_{a+b}$, respectively, $(a, b \in G)$.

Since the order $|G|$ is odd, the pair $(-a+b, a+b)$ runs over all the elements in $G \times G$. Therefore the system of equations

$$u_{-a+b}v_{a+b} = u'_{-a+b}v'_{a+b} \quad (a, b \in G)$$

implies $u_a/u'_a = u_b/u'_b$, $v_a/v'_a = v_b/v'_b$ $(a, b \in G)$. Namely the point $u = (u_a)_G$ and $v = (v_a)_G$ in (2) are uniquely determined by x and y as points in P_G.

Received February 27, 1968.