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A CHARACTERIZATION OF THE ZASSENHAUS GROUPS

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Introduction

A doubly transitive permutation group $\mathfrak{G}$ on the set of symbols Ω is called a Zassenhaus group if $\mathfrak{G}$ satisfies the following condition: the identity is the only element leaving three distinct symbols fixed.

The Zassenhaus groups were classified by H. Zassenhaus [14], W. Feit [3], N. Ito [7], and M. Suzuki [9]. There have been several characterizations of the Zassenhaus groups. Namely M. Suzuki [10] has proved that if a non abelian simple group $\mathfrak{G}$ has a non-trivial partition then $\mathfrak{G}$ is isomorphic with one of the groups $\mathrm{PSL}(2,q)$ or $\mathbf{Sz}(2^n)$. Since each of the groups $\mathrm{PSL}(2,q)$, $\mathbf{Sz}(2^n)$ has a non-trivial partition, a theorem of Suzuki characterizes them.

In this paper we shall characterize the Zassenhaus groups as permutation groups by a property of the centralizer of their involutions.

Let $\mathfrak{G}$ be a finite permutation group on a set of n symbols $\Omega=\{1, 2, \cdots n\}$. For every $i(0\leq i\leq n)$, we define a subset $\mathfrak{C}_i$ of $\mathfrak{G}$ in the following way:

$$\mathfrak{C}_i=\{G\in\mathfrak{G}\,|\,G \text{ leaves exactly } i \text{ distinct symbols fixed}\}.$$

Clearly each $\mathfrak{C}_i$ is a union of some conjugate classes of $\mathfrak{G}$. In particular $\mathfrak{C}_n=\{1\}$. A subset $\mathfrak{C}_i$ may be empty for some i. We shall set a following condition:

(c_i) there exists an involution $I^{(i)}\in\mathfrak{C}_i$ such that the centralizer $\mathfrak{C}_{\mathfrak{G}}(I^{(i)})$ of $I^{(i)}$ in $\mathfrak{G}$ is contained in $\mathfrak{C}_i\cup\{1\}$.

It is easy to see that every conjugate element J of $I^{(i)}$ has the same property as $I^{(i)}$. As a matter of fact, the linear fractional groups $\mathrm{PSL}(2,q)$ and Suzuki's simple groups $\mathbf{Sz}(2^m)$ satisfy one of the conditions (c_0), (c_1) or

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