Mamoru K.
Nagoya Math. J.
Vol. 33 (1968) 21-52

# ON THE SINGULARITY OF GREEN FUNCTIONS IN MARKOV PROCESSES

MAMORU KANDA

## §0. Introduction

In the previous paper [6] we have discussed Markov processes in $R^d$ with the Green function $G(x, y)$ satisfying $C_1 \frac{1}{|x-y|^{d-\alpha}} \leq G(x, y) \leq C_2 \frac{1}{|x-y|^{d-\alpha}}$ ($0 < \alpha \leqq 2$, $C_1 < C_2$ are positive constants), and showed that the regular points of its process are the same as those of $\alpha$-stable process. The present article is closely related to the previous one. We shall discuss several properties of Markov process, including those of regular points, which are sharply influenced by the singularity of the Green function at diagonal set. The singularity we will be concerned with is more general than that of previous one, but it is closely related to that of Riesz kernel. Then, our results may be considered as a generalization of the facts which appear in the relation between the Riesz kernel and stable process. For this purpose potential representation of the hitting probability plays an important role, which we shall show under certain uniform condition about singularity of the Green function instead of duality condition.

The author wishes his thanks to Prof. H. Kunita for his valuable suggestions.

## §1. Notations and main results

Let $R^d (d \geqq 3)$ be the $d$-dimensional Euclidean space and $\Omega$ be a domain in it. Let $\infty$ be adjoined to $\Omega$ and $\Omega \cup \{\infty\}$ be its one-point compactification. We denote by $\mathscr{B}$ the topological Borel field on $\Omega \cup \{\infty\}$. We introduce several spaces of functions on $\Omega$; $B_K$ = *the space of bounded $\mathscr{B}$-measurable functions of compact support*, $C_K$ = *the space of continuous functions of compact support in $\Omega$ and $C_0$ = the space of continuous functions vanishing at infinity.*

An extended real-valued function $G(x, y)$ on $\Omega \times \Omega$ is said to be a *kernel* on $\Omega$, if it is non-negative, continuous except at the diagonal set on $\Omega \times \Omega$.

Received Nov. 10, 1967.