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ON MULTIVARIATE WIDE-SENSE MARKOV PROCESSES*

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1. Introduction: The idea of multivariate wide-sense Markov processes has been recently used by F.J. Beutler [1]. In his paper, he shows that the solution of a linear vector stochastic differential equation in a wide-sense Markov process. We obtain here a characterization of such processes and as its consequence obtain the conditions under which it satisfies Beutler's equation. Furthermore, in stationary Gaussian case we show that these are precisely stationary Gaussian Markov processes studied by J. Doob [5].

In their remarkable papers, T. Hida [6] and H. Cramér [2], [3] have studied the representation of a purely non-deterministic (not necessarily stationary) second order processes. We obtain such a representation for wide-sense Markov processes directly, by using their theory. The interesting part of our representation is that we are able to show that the multiplicity of q-dimensional wide-sense Markov processes does not exceed q, as, in general, even one-dimensional (not necessarily stationary) processes could have infinite multiplicity (see H. Cramér [2] and T. Hida [6]). We also show that the kernel splits (see Theorem 6. 1). As a consequence of this, we obtain the classical representation of Doob [5].

The paper is divided into 7 sections. The next section is devoted to the introduction of terminology and notation used in the rest of the paper.

2. Direct-product Hilbert-spaces: In this section we want to introduce the idea of direct-product Hilbert-spaces as in [10]. If H is a Hilbert-space we shall mean by $H^{(q)}$ the space of all vectors $\underline{h} = (h_1, h_2, \cdots, h_q)$ where for each i, $h_i \in H$. In $H^{(q)}$ is introduced a norm $|||\underline{h}||| = \sqrt{\sum_1^q \| h_i \|_H^2}$ and an inner product given by the Gramian matrix $[h, h^*] = \{< h_i, h_j >_H\}$.

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