A ring \( R \) (with identity) is semi-primary if it contains a nilpotent ideal \( N \) with \( R/N \) semi-simple with minimum condition. \( R \) is called a left \( QF-3 \) ring if it contains a faithful projective injective left ideal. If \( R \) is semi-primary and left \( QF-3 \), then there is a faithful projective injective left ideal of \( R \) which is a direct summand of every faithful left \( R \)-module \([5]\), in agreement with the definition of \( QF-3 \) algebra given by R.M. Thrall \([6]\).

Let \( Q(M) \) denote the injective envelope of a (left) \( R \)-module \( M \). We call \( R \) left \( QF-3^+ \) if \( Q(R) \) is projective. J.P. Jans showed that among rings with minimum condition on left ideals, the classes of \( QF-3 \) and \( QF-3^+ \) rings coincide \([5]\).

In this note we determine the class of semi-primary rings in which the notions of \( QF-3 \) and \( QF-3^+ \) coincide. Next, we show that the characterization of \( QF-3^+ \) rings given by Wu, Mochizuki, and Jans \([7]\) for rings with the property that direct products of projective modules are projective, can be used to characterize semi-primary \( QF-3 \) rings. Finally, we give some results relating the notions of torsionless and torsion-free modules as defined by H. Bass \([1]\) and A.W. Goldie \([3]\). In particular we show that if \( R \) is semi-primary, these notions coincide if and only if \( R \) is left \( QF-3 \) and has zero left singular ideal.

S. Eilenberg has given the following characterization of projective modules for semi-primary rings \([2]\).

**Proposition 1.** If \( R \) is semi-primary and \( P \) is a projective \( R \)-module, then \( P = \oplus \sum P_\ast \) where each \( P_\ast \) is isomorphic to an indecomposable direct summand of \( nR \).

**Proposition 2.** If \( R \) is semi-primary then \( R \) is left \( QF-3^+ \) if and only if \( R \) is left \( QF-3 \) and the left socle of \( R \) is the direct sum of a finite number of simple left ideals of \( R \).

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