HOMOTOPY GROUPS OF COMPACT LIE GROUPS E_6, E_7 AND E_8

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§ 1. Introduction

Let G be a simple, connected, compact and simply-connected Lie group. If k is the field with characteristic zero, then the algebra of cohomology $H^*(G\,;k)$ is the exterior algebra generated by the elements $x_1, \cdots, x_l$ of the odd dimension $n_1, \cdots, n_l$; the integer l is the rank of G and $n = \sum_{i=1}^{l} n_i$ is the dimension of G. Let X be the direct product of spheres of dimension $n_1, \cdots, n_l$, then there exists a continuous map $f: G \longrightarrow X$ which induces isomorphisms of $H^i(X\,;k)$ to $H^i(G\,;k)$ for all i (cf. [8]). From this we deduce by Serre's C-theory [8] that $f_*: \pi_i(G) \longrightarrow \pi_i(X)$ are C-isomorphisms for all i, where C is the class of finite abelian groups. Therefore the rank of $\pi_q(G)$ is equal to the number of such i that n_i is equal to q, and particularly if q is even, then $\pi_q(G)$ is finite. It is a classical fact that $\pi_2(G) = 0$ and $\pi_3(G) = Z$.

According to Bott-Samelson [6] ;

$$\begin{array}{lll} \pi_i(E_6) = 0 & \text{for } 4 \leqslant i \leqslant 8\,, & \pi_9(E_6) = Z, \\ \pi_i(E_7) = 0 & \text{for } 4 \leqslant i \leqslant 10, & \pi_{11}(E_7) = Z, \\ \pi_i(E_8) = 0 & \text{for } 4 \leqslant i \leqslant 14, & \pi_{15}(E_8) = Z. \end{array}$$

where E_6, E_7 and E_8 are compact exceptional Lie groups.

In this paper, using the killing method we compute the 2-components of homotopy group $\pi_j(G)$, where $G = E_6, E_7$ and E_8. The resuls are stated as follows;

j	$4 \leqslant j \leqslant 14$	15	16	17	18	19	20	21	22	23
$\pi_j(E_8 : 2)$	0	Z	Z_2	Z_2	Z_8	0	0	Z_2	0	$Z + Z_2$

j	24	25	26	27	28
$\pi_j(E_8 : 2)$	$Z_2 + Z_2$	Z_2	0	Z	0

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