PROLONGATIONS OF G-STRUCTURES
TO TANGENT BUNDLES

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To Professor Kiyoshi Noshiro on the occasion of his 60th birthday.

§ Introduction and notations.

The purpose of the present paper is to study the prolongations of $G$-structures on a manifold $M$ to its tangent bundle $T(M)$, $G$ being a Lie subgroup of $GL(n, R)$ with $n = \dim M$. Recently, K. Yano and S. Kobayashi [9] studied the prolongations of tensor fields on $M$ to $T(M)$ and they proposed the following question: Is it possible to associate with each $G$-structure on $M$ a naturally induced $G'$-structure on $T(M)$, where $G'$ is a certain subgroup of $GL(2n, R)$? In this paper we give an answer to this question and we shall show that the prolongations of some special tensor fields by Yano-Kobayashi — for instance, the prolongations of almost complex structures — are derived naturally by our prolongations of the classical $G$-structures. On the other hand, S. Sasaki [5] studied a prolongation of Riemannian metrics on $M$ to a Riemannian metric on $T(M)$, while the prolongation of a (positive definite) Riemannian metric due to Yano-Kobayashi is always pseudo-Riemannian on $T(M)$ but never Riemannian. We shall clarify the circumstances for this difference and give the reason why the one is positive definite Riemannian and the other is not.

The crucial starting point for our study is the following simple fact (§ 1): The tangent bundle (space) $T(R^n)$ of the $n$-dimensional real euclidean space $R^n$ is also a vector space and the tangent bundle $T(GL(n, R))$ of the general linear group can be identified to a subgroup of $GL(2n, R)$, the tangent bundle $T(G)$ of a Lie group $G$ being a Lie group by the natural group multiplication. From this fact we can show that, if we denote by $F(M)$ the bundle of frames of $M$, $T(F(M))$ can be imbedded canonically into $F(T(M))$ (§ 2). Using this imbedding and the above identification of $T(GL(n, R))$ to a subgroup of $GL(2n, R)$, we can associate with each $G$-