ON QUASI-LINEAR PARABOLIC EQUATIONS

YOSHIAKI IKEDA*)

To Professor Kiyoshi Noshiro on the occasion of his 60th birthday.

§1 Introduction

In this paper we consider the following quasi-linear parabolic equations

$$(1.1) \qquad Lu = u_t - \text{div } A(x, t, u, u_x) + B(x, t, u, u_x) = 0 ,$$

where A is a given vector function of the variables x, t, u, u_x, and B is a given scalar function of the some variables. We assume that they are difined in the rectangle

$$R = \{(x, t) \in E^{n+1} \mid x = (x_1, \ldots\ldots, x_n) \in E^n, |x_i| < 2r, 0 < t < 2r^2\}$$
$$= Q_{2r} \times (0, 2r^2), \text{ where } Q_{2r} = \{x \mid |x_i| < 2r\} .$$

Moreover we assume that

$$(1.2) \qquad \begin{cases} |A(x, t, u, p)| \leqslant M|p| + c(x, t)|u| + e(x, t) \\ |B(x, t, u, p)| \leqslant b(x, t)|p| + d(x, t)|u| + f(x, t) \\ pA(x, t, u, p) \geqslant \lambda |p|^2 - d(x, t)|u|^2 - g(x, t) \end{cases}$$

for any real vector $p = (p_1, \ldots\ldots, p_n)$. Here M and λ are positive constants, and b, c, d, e, f and g are non-negative functions of the variables x, t such that

$$(1.3) \qquad \begin{cases} b, c, e \in L^\infty[0, 2r^2; L^{n+\varepsilon}(Q_{2r})], \quad d, f, g \in L^\infty[0, 2r^2; L^{\frac{n+\varepsilon}{2}}(Q_{2r})] \\ \text{for arbitrary } \varepsilon > 0 \text{ and} \\ \max\limits_{0<t<2r^2} \|d\|_{n+\varepsilon}(t) + \max\limits_{0<t<2r^2} \|c\|_{n+\varepsilon}(t) + \max\limits_{0<t<2r^2} \|e\|_{n+\varepsilon}(t) + \max\limits_{0<t<2r^2} \|d\|_{\frac{n+\varepsilon}{2}}(t) \\ + \max\limits_{0<t<2r^2} \|f\|_{\frac{n+\varepsilon}{2}}(t) + \max\limits_{0<t<2r^2} \|g\|_{\frac{n+\varepsilon}{2}}(t) < M, \end{cases}$$

where $\|w\|_p(t) = \left(\int_{Q_{2r}} |w|^p dx\right)^{1/p}$

We denote by $L^q[0, 2r^2; L^p(Q_{2r})]$ the space of function $\varphi(x, t)$ with the following properties:

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