ON THE BOUNDARY BEHAVIOR OF CONFORMAL MAPS*)

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To Professor Kiyoshi Noshiro on his 60th birthday

INTRODUCTION. Suppose Ω is a simply connected domain which is mapped conformally onto a disk. A much studied problem is the behavior of the mapping function at an accessible boundary point P of Ω, in particular the question, under what conditions the map is "conformal" at such a point (a) in the sense that angles are preserved as P is approached from Ω ("semi-conformality" at P) and (b) the dilatation at P is finite and positive. In his fundamental paper [8] in 1936, A. Ostrowski established a necessary and sufficient condition (depending on the geometry of the domain only) for the validity of the first property which subsumes all previous results and establishes a definitive solution of this problem. There has been extensive work on problem (b) (see [7] and [5, Chapters IV and VI]); in particular, a number of criteria have been obtained for the existence of the "angular derivative" of the mapping function and of the derivative for "unrestricted approach" (see [10]). In establishing a connection between angular and unrestricted derivatives a second theorem in [8], Ostrowski's "Faltensatz" plays an important role. The "Faltensatz" is a generalization of an earlier result [10] of the author for Jordan domains, which he used to deduce the existence of the unrestricted derivative from that of the angular derivative.

Ostrowski's proof of his two theorems are based on the systematic use of the harmonic measure and requires a rather extensive study of its properties. Later on another proof utilizing an entirely different method (Carathéodory's convergence theorem on the mapping functions of domains tending to a kernel) was given by J. Lelong Ferrand [4]; [5, Chapter IV]. In the present paper we use a method of J. Wolff [11], abstracted in our Lemma 1, to give new

Received July 29, 1966.
Received August 15, 1966.
*) The preparation of this paper was sponsored (in part) by the Office of Naval Research under contract Nonr-2216(28) (NR-043-332) with the University of California.