ON A CONJECTURE OF J.S. FRAME

NOBORU ITO*

To Professor Kiyoshi Noshiro on His Sixtieth Birthday

Let $\mathfrak{G}$ be a transitive group of degree n, and let $\mathfrak{G}_1$ be the stabilizer of a symbol in $\mathfrak{G}$. Then we owe to J.S. Frame the following remarkable relations between the lengths n_i of the orbits of $\mathfrak{G}_1$ and the degrees f_i of the absolutely irreducible components of the permutation matrix representation $\mathfrak{G}^*$ of $\mathfrak{G}$:

(A) If the irreducible constituents of $\mathfrak{G}^*$ are all different, then the rational number

$$F=n^{k-2}\prod_{i=1}^{k} n_i/f_i$$

is an integer, where k is the number of the orbits of $\mathfrak{G}_1$.

(C) If the irreducible constituents of $\mathfrak{G}^*$ all have rational characters, then F is a square.

Further J.S. Frame made the following conjecture ([1]):

(B) If the k numbers n_i are all different, then F is a square.

(B) is true for $k\leqq 3$ ([3], §30).

Now the purpose of this short note is to show that (B) is not true in general for $k=4$.

Let $LF_r(q)$ be the r-dimensional projective special linear group over the field of q elements such that $p=\dfrac{q^r-1}{q-1}$ is a prime and r is odd. Let $V_r(q)$ and $W_r(q)$ be the r-dimensional spaces of column and row vectors over the field of q elements, respectively. Let V and W be the set of one-dimensional subspaces of $V_r(q)$ and $W_r(q)$, respectively. $\left\langle\begin{matrix}x_1\\ \vdots\\ x_r\end{matrix}\right\rangle\in V$ and $<y_1, \ldots\ldots, y_r>\in W$ denote the one-dimensional subspaces generated by $\begin{pmatrix}x_1\\ \vdots\\ x_r\end{pmatrix}\in V_r(q)$ and $(y_1, \ldots\ldots, y_r)$

* This research was partially supported by National Science Foundation Grand GP-5276