ON LOCAL MAXIMALITY FOR THE COEFFICIENT a_6

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Dedicated to Professor K. Noshiro on his 60th birthday

1. Recently a number of authors have studied the application of Grunsky's coefficient inequalities to the study of the Bieberbach conjecture for the class of normalized regular univalent functions $f(z)$ in the unit circle $|z|<1$

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n .$$

Charzynski and Schiffer [2] applied this result to give an elementary proof of the inequality $|a_4|\leqq 4$. One of the present authors [8] proved that if a_2 is real non-negative then $\Re\, a_6\leqq 6$. A natural first step in the study of the inequality for a coefficient is to prove local maximality for a_2 near to 2. Bombieri [1] announced that he had proved

$$\Re\, a_6 \leqq 6-A(2-\Re\, a_2)$$

for $A>0$, $\Re\, a_2$ sufficiently near to 2. As yet to our knowledge no complete account of his result has appeared. One of the present authors has shown [7] that in many cases the Area Principle is more effective than Grunsky's method. In the present instance the Area Principle takes the form of an inequality due to Golusin [4]. In this paper we use this inequality to prove the local maximality of $\Re\, a_6$ at the Koebe function. Our theorem implies the result of Bombieri.

During the preparation of this work there appeared a paper by Garabedian, Ross and Schiffer [3] which asserts the local maximality of $\Re\, a_{2n}$, $n=2,3,\ldots\ldots$ at the Koebe function. Further consideration is required to determine its status. In any case it does not appear to include Bombieri's result.

2. Golusin's inequality and Grunsky's inequality.

Let $f(z)$ be a normalized regular function univalent in the unit disc $|z|<1$, whose expansion around $z=0$ is

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