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STOCHASTIC STABILITY OF ANOSOV DIFFEOMORPHISMS

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§ 0. Introduction

R. Bowen [1] introduced the notion of pseudo-orbit for a homeomorphism f of a metric space X as follows: A (double) sequence $\{x_i\}_{i\in Z}$ of points x_i in X is called a δ-pseudo-orbit of f iff

$$d(fx_i, x_{i+1}) \leq \delta$$

for every $i \in Z$, where d denotes the metric in X. We say f is stochastically stable if for every $\varepsilon > 0$ there exists $\delta > 0$ such that every δ-pseudo-orbit $\{x_i\}_{i\in Z}$ of f is ε-traced by some $x \in X$, i.e.,

$$d(f^i x, x_i) \leq \varepsilon$$

for every $i \in Z$. He proved in [1] that if a compact hyperbolic set Λ for a diffeomorphism f of a compact manifold M has local product structure then the restriction $f|\Lambda$ of f to Λ is stochastically stable, using stable and unstable manifolds.

In this paper we prove first that an Anosov diffeomorphism f of a compact manifold M is topologically stable, in the set of all continuous maps of M into M, in a sense (Theorem 1). Next, making use of Theorem 1 we give another proof for Bowen's result, in the case of f an Anosov diffeomorphism (Theorem 2). The idea of this paper is inspired by a result of A. Morimoto [2], which says that a topologically stable homeomorphism f of a manifold M with $\dim M \geq 3$ is stochastically stable. The method of the proof follows that of P. Walters [3].

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§ 1. Preparatory lemmas

M will always denote a compact C^∞ manifold without boundary.

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