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THE FOURTH DIMENSION SUBGROUPS AND POLYNOMIAL MAPS, II

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§ 1. Introduction

In our previous paper [3] we proved the following ([3, Theorem 16]):

THEOREM A. *Let G be a 2-group of class 3. Let G_2 and G/G_2 be direct products of cyclic groups $\langle y_q \rangle$ of order α_q $(1 \leqq q \leqq m)$, and of cyclic groups $\langle h_i \rangle$ of order β_i $(1 \leqq i \leqq n)$ with $\beta_1 \geqq \beta_2 \geqq \cdots \geqq \beta_n$, respectively. Let x_i be representatives of h_i $(1 \leqq i \leqq n)$, and put $x_i^{\beta_i} = y_1^{c_{i1}} y_2^{c_{i2}} \cdots y_m^{c_{im}}$ $(1 \leqq i \leqq n)$, $[x_j, y_s] = y_1^{e_1^{js}} y_2^{e_2^{js}} \cdots y_m^{e_m^{js}}$ $(1 \leqq j \leqq n, 1 \leqq s \leqq m)$. Then a homomorphism $\psi: G_3 \to T$ can be extended to a polynomial map from G to T of degree $\leqq 4$ if and only if there exists an integral solution in the following linear equations of X_{iq} $(1 \leqq i \leqq n, 1 \leqq q \leqq m)$ with coefficients in T:*

$$\sum_{1 \leqq q \leqq m} e_q^{js} \frac{X_{iq}}{(\beta_i, \alpha_q)} = 0 \qquad (1 \leqq i, j \leqq n, 1 \leqq s \leqq m) \tag{I}$$

$$2^{\delta_{ij}} \left[\sum_{1 \leqq q \leqq m} c_{iq} \frac{X_{jq}}{(\beta_j, \alpha_q)} - \left(\frac{\beta_i}{\beta_j} \right) \sum_{1 \leqq q \leqq m} c_{jq} \left\{ \frac{X_{iq}}{(\beta_i, \alpha_q)} + \psi([x_i, y_q]) \right\} \right] = 0 \tag{II}$$
$$(1 \leqq i < j \leqq n),$$

where δ_{ij} is the Kronecker symbol for β_i: i.e. $\delta_{ij} = 1$ or 0 according to $\beta_i = \beta_j$ or $\beta_i > \beta_j$, respectively.

As corollaries we had

COROLLARY 1 ([3, Corollaries 18 and 21]). *If $2 \leqq n \leqq 3$: i.e. the rank of G/G_2 is at most three, then $D_4(G) = G_4$.*

In this paper we discuss the problem in the case $n \geqq 4$. We find out some sufficient conditions for $D_4(G) = G_4$ in the general case $n \geqq 4$, as the case such that the equations (I) and (II) in Theorem A have a

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