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ANALYTIC STRUCTURE OF SCHLÄFLI FUNCTION

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§1. Introduction

In this note it is shown that *Schläfli function can be simply expressed in terms of hyperlogarithmic functions, namely iterated integrals of forms with logarithmic poles in the sense of K. T. Chen* (Theorem 1). It is also discussed *the relation between Schläfli function and hypergeometric ones of Mellin-Sato type* (Theorem 2). From a combinatorial point of view the structure of hyperlogarithmic functions seem very interesting just as the dilog $\int_0^x \log(1-x)/x dx$ (so-called Abel-Rogers function) has played a crucial part in Gelfand-Gabriev-Losik's formula of 1st Pontrjagin classes. See also [3].

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§2. Gauss-Bonnet theorem

Let S^n be a n dimensional unit sphere in $\boldsymbol{R}^{n+1}$ with the standard metric and $S_1, S_2, \cdots, S_{n+1}$ be $(n+1)$ hyperplanes in $\boldsymbol{R}^{n+1}$ through the origin which are in general position. Let

$$(2.1) \qquad S_j : f_j = 0$$

where $f_j = \sum_{\nu=1}^{n+1} u_{j\nu} x_\nu$ with $\sum_{\nu=1}^{n+1} u_{j\nu}^2 = 1$. The set of all points of S^n satisfying the inequalities

$$(2.2) \qquad f_1 \geqq 0, \cdots, f_{n+1} \geqq 0$$

form a n dimensional spherical simplex denoted by Δ. We denote by

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