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ON p-ADIC L-FUNCTIONS AND CYCLOTOMIC FIELDS. II

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1. Introduction

Let p be a prime. If one adjoins to $\boldsymbol{Q}$ all p^n-th roots of unity for $n = 1, 2, 3, \cdots$, then the resulting field will contain a unique subfield $\boldsymbol{Q}_\infty$ such that $\boldsymbol{Q}_\infty$ is a Galois extension of $\boldsymbol{Q}$ with $\mathrm{Gal}(\boldsymbol{Q}_\infty/\boldsymbol{Q}) \cong \boldsymbol{Z}_p$, the additive group of p-adic integers. We will denote $\mathrm{Gal}(\boldsymbol{Q}_\infty/\boldsymbol{Q})$ by Γ. In a previous paper [6], we discussed a conjecture relating p-adic L-functions to certain arithmetically defined representation spaces for Γ. Now by using some results of Iwasawa, one can reformulate that conjecture in terms of certain other representation spaces for Γ. This new conjecture, which we believe may be more susceptible to generalization, will be stated below.

Let $\boldsymbol{Q}_p$ be the field of p-adic numbers and let Ω_p be an algebraic closure of $\boldsymbol{Q}_p$. Let ψ be an even primitive Dirichlet character which takes its values in Ω_p and which is of the first kind (this means that the conductor of ψ is not divisible by p^2 if p is odd or by 8 if $p = 2$). Let K be the cyclic extension of $\boldsymbol{Q}$ associated to ψ by class field theory and let $K_\infty = K\boldsymbol{Q}_\infty$, the cyclotomic $\boldsymbol{Z}_p$-extension of K. Let M_∞ denote the maximal abelian pro-p-extension of K_∞ in which only primes of K_∞ dividing p are ramified. (We also allow the infinite primes to be ramified, although this could happen only if $p = 2$). Now Γ can be identified in a natural way with $\mathrm{Gal}(K_\infty/K)$ and, by means of this identification, we can consider $\mathrm{Gal}(M_\infty/K_\infty)$ as a Γ-module. One can then define quite simply a certain representation space W_ψ for Γ over Ω_p (see Section 2).

In [11], Leopoldt and Kubota have constructed a p-adic L-function $L_p(s, \psi)$ for every primitive even Dirichlet character ψ. This function is defined for all $s \in \boldsymbol{Z}_p$ (except for $s = 1$ if ψ is the principal character ψ_0) and takes its values in Ω_p. Now it follows easily from a result of

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