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NOTE ON CLASS NUMBER FACTORS AND PRIME DECOMPOSITIONS

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Introduction

Let K be a Galois extension of an algebraic number field k of finite degree with Galois group g, $\mathfrak{D}$ be a congruent ideal class group of K, and M be the class field over K corresponding to $\mathfrak{D}$. Assume that M is normal over k. Then g acts on $\mathfrak{D}$ as a group of automorphisms. Donote by I_g the augmentation ideal of the group ring $\boldsymbol{Z}_g$ over the ring of integers $\boldsymbol{Z}$. Then we have a sequence $\mathfrak{D} \supset I_g\mathfrak{D} \supset I_g^2\mathfrak{D} \supset \cdots$ and a sequence of the corresponding class fields $K = K_{M/k}^{(0)} \subset K_{M/k}^{(1)} \subset K_{M/k}^{(2)} \subset \cdots$. We call $K_{M/k}^{(i)}$ the *i-th central class field of K in M with respect to k.* We put simply $K^{(i)} = K_{M/k}^{(i)}$, when it is not in danger of confusion.

In the previous paper [10], we have shown that the Galois group $G(K^{(i+1)}/K^{(i)})$ is isomorphic to a factor group of $G(K^{(1)}/K)$ or of slightly modified group of $G(K^{(1)}/K)$ when K is non-cyclic over k.

In the present paper we apply the above result firstly to the case where K is cyclic over k and we have more explicit structure of $G(K^{(i+1)}/K^{(i)})$. In fact we have a formula of the extension degree of $K^{(i+1)}/K^{(i)}$ in §2, which generalize the genus formula in [8] when K is cyclic over k. Furthermore in §3 we express the structure of $G(K^{(i+1)}/K^{(i)})$ by using "Auflösung" characters of H. W. Leopoldt [19], when the ground field k is the rational number field $\boldsymbol{Q}$.

Secondly we study on prime decompositions in $K^{(i+1)}/K^{(i)}$ and in §5 we have explicit criteria of prime decompositions for some non-abelian extensions. As a special case we have a new expression of the reciprocity of the biquadratic residue symbol. §2 and §3 are unnecessary to the argument of §4 and §5.

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