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A CLASSIFICATION OF IRREDUCIBLE PREHOMOGENEOUS VECTOR SPACES AND THEIR RELATIVE INVARIANTS

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Introduction

Let G be a connected linear algebraic group, and ρ a rational representation of G on a finite-dimensional vector space V, all defined over the complex number field C.

We call such a triplet (G, ρ, V) a *prehomogeneous vector space* if V has a Zariski-dense G-orbit. The main purpose of this paper is to classify all prehomogeneous vector spaces when ρ is irreducible, and to investigate their relative invariants and the regularity.

This paper consists of the following eight sections.

§ 1. Preliminaries
§ 2. Castling transforms
§ 3. Classification of reduced triplets (G, ρ, V) satisfying $\dim G \geqq \dim V$
§ 4. Relative invariants and the regularity
§ 5. The prehomogeneity and relative invariants of reduced triplets obtained in § 3
§ 6. The semi-simple case
§ 7. Table of reduced irreducible prehomogeneous vector spaces
§ 8. Prehomogeneous vector spaces with finitely many orbits

We now make a brief survey of this paper. For the convenience of the reader, we shall review, at the beginning of § 1, basic facts about complex simple Lie algebras, especially their irreducible representations and their classification. Then we shall construct a simple Lie algebra of each type and calculate its representation degrees which will be used in § 3. We shall introduce in § 2 an important notion of castling transform, which is an irreducible prehomogeneous vector space obtained from

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