DIFFERENTIAL EQUATIONS AND AN ANALOG OF THE
PALEY-WIENER THEOREM FOR LINEAR
SEMISIMPLE LIE GROUPS

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§ 1. Introduction

Let $G$ be a noncompact linear semisimple Lie group. Fix $G = KAN$ an Iwasawa decomposition of $G$. That is, $K$ is a maximal compact subgroup of $G$, $A$ is a vector subgroup with $AdA$ consisting of semisimple transformations and $A$ normalizes $N$, a simply connected nilpotent subgroup of $G$. Let $M'$ denote the normalizer of $A$ in $K$, $M$ the centralizer of $A$ in $K$, and $W = M'/M$ the restricted Weyl group of $G$. Fix $\theta$ a Cartan involution of $G$ which leaves every element of $K$ fixed and set $\overline{N} = \theta N$. We denote the Lie algebras of $G, K, A, N, \overline{N}$, and $M$ respectively by $\mathfrak{g}, \mathfrak{k}, \mathfrak{a}, \mathfrak{n}, \mathfrak{m},$ and $\mathfrak{m}$ respectively.

For $g \in G$ set $g = K(g) \exp H(g) n(g)$ where $K(g) \in K$, $H(g) \in \mathfrak{k}$, and $n(g) \in N$ and $\exp : \mathfrak{k} \rightarrow A$ is an isomorphism from $\mathfrak{k}$ to $A$ with inverse log. Recall that $\lambda \in \mathfrak{a}^*$ is called a root if $\mathfrak{g}_\lambda = \{X \in \mathfrak{g} : [H, X] = \lambda(H)X$ for all $H \in \mathfrak{k}\} \neq \{0\}$ and $\lambda$ is a positive root if $\mathfrak{g}_\lambda \subseteq \mathfrak{r}$. Let $P$ denote the set of all positive roots and let $L$ be the semilattice of all elements of $\mathfrak{a}^*$ of the form $\sum_{i \in P} c_i \lambda$ and $c_i$ is a nonnegative integer.

Let $V$ be a finite dimensional vector space and let $K$ act on $V$ via the double representation $\tau$. That is, for $v \in V$ and $k_1, k_2 \in K$

$$\tau(k_1, k_2) : v \rightarrow \tau(k_1) \cdot v \cdot \tau(k_2)^{-1}.$$ Consider the $C^\infty$ functions $f : G \rightarrow V$ for which $f(k_1 g k_2) = \tau(k_1) f(g) \tau(k_2)$ $(k_1, k_2 \in K)$. We denote these functions by $C^\infty(G, \tau)$ and we denote the $C^\infty$-functions with compact support by $C_c^\infty(G, \tau)$ and the Schwartz functions in $C^\infty(G, \tau)$ by $\mathcal{S}(G, \tau)$.

Consider $f \in \mathcal{S}(G, \tau)$ and for $\nu \in \mathfrak{a}_c^*$ let $m \in M$ set

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