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A NOTE ON HOLOMORPHIC MATRIC AUTOMORPHIC FACTORS WITH RESPECT TO A LATTICE IN A COMPLEX VECTOR SPACE

Dedicated to the memory of Taira Honda

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1. A holomorphic $n \times n$-matric automorphic factor with respect to a lattice L in C^g means a system of holomorphic $n \times n$-matrices $\{\mu_\alpha(z) \mid \alpha \in L\}$ such that

$$(1) \qquad \det \mu_\alpha(z) \neq 0 \qquad \text{everywhere on } C^g ,$$

$$(2) \qquad \mu_{\alpha+\beta}(z) = \mu_\alpha(z+\beta)\mu_\beta(z) \qquad (\alpha, \beta \in L) .$$

This is nothing else than the condition of a group action of L on $C^g \times C^n$;

$$(z, u) \longrightarrow (z + \alpha, \mu_\alpha(z)u) \qquad (\alpha \in L) .$$

The quotient $E_\mu = C^g \times C^n/L$ by this group action of L is a holomorphic vector bundle of rank n over the complex torus C^g/L. Holomorphic vector bundles over the complex torus C^g/L are always constructed by this way, since holomorphic vector bundles over C^g are trivial.

Denoting

$$\omega_\alpha(z) = \mu_\alpha(z)^{-1}d\mu_\alpha(z) \qquad (\alpha \in L)$$

we get a system of $n \times n$-matric connections satisfying

$$(3) \qquad d\omega_\alpha(z) + \omega_\alpha(z) \wedge \omega_\alpha(z) = 0 ,$$

$$(4) \qquad \omega_{\alpha+\beta}(z) = \omega_\alpha(z) + \mu_\alpha(z)^{-1}\omega_\beta(z+\alpha)\mu_\alpha(z) \qquad (\alpha, \beta \in L) .$$

In the present short note we shall characterize matric automorphic factors $\{\mu_\alpha(z) \mid \alpha \in L\}$ such that

i) the associated vector bundle E_μ is simple and ii)

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