T. Fukuda
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DEFORMATIONS OF REAL ANALYTIC FUNCTIONS AND THE NATURAL STRATIFICATION OF THE SPACE OF REAL ANALYTIC FUNCTIONS

TAKUO FUKUDA

0. Introduction.

Let A be a real analytic set, M be a compact real analytic manifold and $f: A \times M \to \boldsymbol{R}$ be a real analytic function. Then we have a family of real analytic functions $f_a, a \in A$, on M defined by $f_a(X) = f(a, x)$.

Two functions f_a and f_b are said to be topologically equivalent if there exist homeomorphisms h_1 of M and h_2 of $\boldsymbol{R}$ such that $h_2 \circ f_a = f_b \circ h_1$.

The purpose of the present paper is to prove the following

THEOREM 1. *There is a Whitney stratification of A satisfying the following properties*:

(i) *Each stratum is a smooth subanalytic subset of A. (For the subanalycity, see* § 3.)

(ii) *For any two points a and b belonging to the same stratum, the corresponding functions f_a and f_b are topologically equivalent.*

COMMENT 1. By Theorem 1, we can see that any analytic deformation of a real analytic function on M contains locally only a finite number of topological types of functions: An analytic deformation of an analytic function $g: M \to \boldsymbol{R}$ is an analytic function $f: U \times M \to \boldsymbol{R}$, U being an open set of $\boldsymbol{R}^n$, or the family $\{f_a\}$, $a \in U$, of real analytic functions on M defined by $f_a(x) = f(a, x)$ such that $f_o = g$ where o is the origin of $\boldsymbol{R}^n$. Then the above statement means that there is a neighborhood $U(o)$ of o in U such that the number of the topological equivalence classes of functions f_a, $a \in U(o)$, is finite. This property holds even for deformations of a function of infinite codimension. This is a special phenomena for analytic deformations. In fact, it is known that there is a

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