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A FEW THEOREMS ON COMPLETION OF EXCELLENT RINGS[1]

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Introduction.

In [2], chap. IV, 2^{me} partie, (7.4.8), Grothendieck considered the following problem: is any $\mathfrak{m}$-adic completion of an excellent ring A also excellent?

In [8] I proved that, if A is an algebra of finite type over an arbitrary field k, the answer is positive.

When A is finitely generated over an excellent Dedekind domain C there is a partial positive answer in [5] for $C =$ Dedekind domain of characteristic 0 and with perfect residue fields.

In the present paper I can remove the condition on the residue fields and show that, if A is an algebra of finite type over an excellent ring C of characteristic 0 and dimension 1, then $(A, \mathfrak{m})^\wedge$ is excellent for every ideal $\mathfrak{m}$ of A.

The result follows from a proper use of jacobian criteria of regularity and from a theorem I can prove here on the closedness of singular loci. Precisely I can show that, if R is $\mathfrak{m}$-adically separated and complete, $R/\mathfrak{m}$ is excellent and the singular locus of every finite R-algebra is closed, then the formal fibers of R are geometrically regular. Such result allows us also to prove excellent property for some new class of completions of excellent rings; for instance, if A is a two-dimensional Nagata ring, all $\mathfrak{m}$-adic completions of A are excellent rings.

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Terminology will be the same as in [8] and [3]. Unique exception:

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