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PERTURBED BILLIARD SYSTEMS, I. THE ERGODICITY OF THE MOTION OF A PARTICLE IN A COMPOUND CENTRAL FIELD

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§ 1. Introduction

The ergodicity of classical dynamical systems which appear really in the statistical mechanics was discussed by Ya. G. Sinai [9]. He announced that the dynamical system of particles with central potential of special type in a rectangular box is ergodic. However no proofs have been given yet. Sinai [11] has given a proof of the ergodicity of a simple one-particle model which is called a Sinai billiard system.

In this article, the author will show the ergodicity of the dynamical system of a particle in a compound central field in 2-dimensional torus (see. § 10). For such a purpose, a new class of transformations, which are called perturbed billiard transformations will be introduced. Let T_* be a perturbed billiard transformation which satisfies the assumptions (H–1), (H–2) and (H–3) (see § 3). Then T_* is expressed in the form

$$T_* = T_1 T \tag{1.1}$$

where T is a Sinai billiard transformation and T_1 is a rotation such that

$$T_1(\iota, r, \varphi) = (\iota, r + H_\iota(\varphi), \varphi) . \tag{1.2}$$

In Theorem 1, 2 and 3, the following assertions will be shown.

(a) There exists a generator $\alpha^{(c)}$ with finite entropy.

(b) Every element of the partition $\zeta^{(c)} = \bigvee_{i=0}^{\infty} T_*^i \alpha^{(c)}$ (resp. $\zeta^{(e)} = \bigvee_{i=-1}^{-\infty} T_*^i \alpha^{(c)}$) is a connected decreasing (resp. increasing) curve.

(c) $T_*^{-1}\zeta^{(c)} > \zeta^{(c)}$, $T_*\zeta^{(e)} > \zeta^{(e)}$,

$$\bigvee_{i=-\infty}^{\infty} T_*^i \zeta^{(c)} = \bigvee_{i=-\infty}^{\infty} T_*^i \zeta^{(e)} = \varepsilon ,$$

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