TWO THEOREMS ON EXCELLENT RINGS[1]

SILVIO GRECO

Let $f: A \to B$ be a homomorphism of commutative noetherian rings. The main results of this paper are:

(a) Assume f is finite and induces a surjective map on the spectra. Then if B is quasi-excellent A is quasi-excellent and is excellent if it is universally catenarian (Th. 3.1)[2]; and

(b) If f is absolutely flat and A is excellent then B is excellent (Th. 5.3). In particular the strict henselization of an excellent local ring is excellent (Cor. 5.6.).

To prove the above we give some more general partial results. For instance in section 1 we study the finite descent for certain P-homomorphisms and P-rings, and in section 2 we show that several properties related with the openness of loci (such as property J2 of excellent rings) descend by surjective scheme morphisms locally of finite type. Result (a) is given in section 3 together with some applications, while in section 4 we show with a counterexample that the assumption "f finite" cannot be replaced by "f of finite type". Section 5 contains result (b), which generalizes the main statements of [5]. This is possible by a theorem of André [1] and a new proof for the chain condition.

The author wishes to thank Prof. H. Matsumura for encouraging him to write this paper and for several useful conversations on its content.

Foreword. All rings are assumed to be commutative and noetherian. We use freely the notations and the definitions of [9] and [7].

1. Finite descent for P-homomorphisms and P-rings.

Let A be an algebra over the field k and let P be any of the fol-

---