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ON CONSTRUCTION OF HOLOMORPHIC CUSP FORMS OF HALF INTEGRAL WEIGHT

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Introduction

In [10], G.Shimura gave a method of constructing holomorphic cusp forms of even integral weight from given forms of half integral weight. In this paper, we try to present an inverse construction. To state our main result, some notational preliminaries are necessary. We denote by $\mathfrak{H}$ the complex upper half plane. Let $x(u, v) = x_1u^2 + x_2uv + x_3v^2$ be an integral binary quadratic form with positive discriminant $d_x = x_2{}^2 - 4x_1x_3$. If $d_x = m^2\,(m > 0)$ is a square, we denote by $C(x)$ the geodesic line with respect to the Poincaré metric on $\mathfrak{H}$ from $(x_2 + m)/2x_3$ to $(x_2 - m)/2x_3$ (if $x_3 = 0$, we understand $C(x)$ to be the geodesic line from $+i\infty$ (resp. x_1/x_2) to x_1/x_2 (resp. $+i\infty$) for $x_2 > 0$ (resp. $x_2 < 0$)). If d_x is not a square and if x_1, x_2 and x_3 have no non-trivial common divisor, let $t_x + u_x\sqrt{d_x} > 1$ be the smallest half-integer solution of the Pell-equation $t^2 - u^2d_x = 1$ and set $\gamma_x = \begin{pmatrix} t_x - x_2u & 2x_1u \\ -2x_3u & t_x + x_2u \end{pmatrix} \in SL_2(\boldsymbol{Z})$. We denote by $C(x)$ any rectifiable curve in $\mathfrak{H}$ from w to $\gamma_x \cdot w$, where w is any point on $\mathfrak{H}$. Finally if x_1, x_2 and x_3 have the greatest common divisor $t > 1$, we put $C(x) = C(x/t)$.

Now, let $f(z)$ be a holomorphic cusp form on the upper half plane which satisfies

$$f(\gamma \cdot z) = \chi^2(d)(cz + d)^{2k} f(z)$$

for any $\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_0(N)$, where χ is a character modulo N and $\Gamma_0(N) = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in SL_2(\boldsymbol{Z}) \,|\, c \equiv 0 \pmod{N}\right\}$. Two integral binary quadratic

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