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HIDA-CRAMÉR MULTIPLICITY THEORY FOR MULTIPLE MARKOV PROCESSES AND GOURSAT REPRESENTATIONS

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I. Introduction.

This work grew out of an attempt to prove the false result that an n-ple Markov process in the sense of Hida (1960) or Lévy (1956a) has multiplicity one. Instead we proved the representation theorem (Theorem III. 1.) that a centered Gaussian process $x(t)$ is n-ple Markov iff it can be written in the form

$$x(t) = \sum_1^n e_i(t)a_i(t) \tag{I.1}$$

where $A(t) = \{a_i(t)\}_{i=1,\cdots,n}$ is a Gaussian martingale with

$$\mathrm{sp}\,\{x(s)\colon s \le t\} \equiv \mathrm{sp}\,\{a_i(s)\colon s \le t \text{ and } 1 \le i \le n\} \tag{I.2}$$

and $A(t)$ and $\{e_i(t)\}$ satisfy some non-degeneracy condition. We also show (Corollary IV. 13.) that for any Gaussian martingale $A(t)$ with simple left innovation spectrum, continuous $e_i(t)$ may be found so that the process $x(t)$ given in (I.1) will satisfy (I.2).

Together these results show that the only restrictions of the possible spectral type of an n-ple Markov process is that it has multiplicity $M \le n$. These constitute our main results on n-ple Markov processes and the remainder of the paper is devoted to studying the implications that a process $x(t)$ admits a "Goursat" representation of the form (I.1).

Section II contains preliminaries on multiplicity theory and Gaussian martingales. In Section III, we prove the basic Theorem III. 1 mentioned above and derive analogous results for the covariance functions. Section IV developes the basic theory of Goursat representations. IV. 3

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