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NEGATIVE VECTOR BUNDLES AND COMPLEX FINSLER STRUCTURES

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1. Introduction

A complex Finsler structure F on a complex manifold M is a function on the tangent bundle $T(M)$ with the following properties. (We denote a point of $T(M)$ symbolically by (z,ζ), where z represents the base coordinate and ζ the fibre coordinate.)

(1.1) F is smooth outside of the zero section of $T(M)$;[1]

(1.2) $F(z,\zeta) \geqq 0$ and $=0$ if and only if $\zeta = 0$;

(1.3) $F(z,\lambda\zeta) = |\lambda|^2 \cdot F(z,\zeta)$ for any complex number λ.

The geometry of complex Finsler structures was first studied by Rizza [8]. In [9], Rund explained the significance of (1.3) in detail and derived the connection coefficients and the equation of the geodesics. In this paper we study complex Finsler structures in holomorphic vector bundles. It is known that a holomorphic vector bundle is ample (in the sense of algebraic geometry) if it admits a hermitian metric of positive curvature.[2] The converse is probably not true in general (except, of course, in the case of line bundles). We prove that a holomorphic vector bundle is negative[3] (i.e., its dual is ample) if and only if it admits a complex Finsler metric of negative curvature, thus making it possible to introduce differential geometric techniques into the study of ample and negative vector bundles.

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1) F will be hermitian if it is smooth everywhere.

2) See Griffiths [3] and Kobayashi-Ochiai [6]. We use the term "ample" in the sense of Hartshorne [4].

3) Our negativity coincides with Grauert's weak negativity [2].