J. D. Donald and F. J. Flanigan
Nagoya Math. J.
Vol. 57 (1975), 107–119

DEFORMATION METHODS AND THE STRONG UNBOUNDED REPRESENTATION TYPE OF p-GROUPS

J. D. DONALD AND F. J. FLANIGAN

Introduction.

A basic problem in the representation theory of a finite group G is the determination of all indecomposable G-modules. Thus, for $G = C(n)$ $=$ a cyclic group of order n over an arbitrary field, the indecomposable representations, finite in number, are known from the theory of a single linear transformation. In 1954 Higman [9] showed that, in sharp contrast to the classical case of characteristic zero, an arbitrary finite group G has indecomposables of arbitrarily high dimension over any field of prime characteristic p iff the p-Sylow subgroup of G is non-cyclic (cf. unbounded representation type [3, p. 431]). Examples published by Heller and Reiner [8] in 1961 indicated that this phenomenon is even more extensive; reinterpreting a result of Dieudonné [4] as classifying the indecomposable modules for a square zero algebra on two generators, they showed that $G = C(p) \times C(p)$ (and therefore many other groups) has infinitely many non-isomorphic indecomposables in every even dimension over an infinite field of characteristic p (cf. strong unbounded representation type). At present, all $C(p) \times C(p)$ indecomposables are known only in the case $p = 2$, the result also being given (essentially) in [4] (cf. also [1], [2], [12]). In particular, the four-group $C(2) \times C(2)$ affords only two (dual) indecomposable representations in each *odd* dimension ≥ 3.

This paper contributes, by way of examples and a suggested technique, to a fuller description of this plethora of G-modules. Our study of the deformation of algebra representations [5], [6], [7], when brought to bear on the Heller-Reiner modules for a non-cyclic abelian p-group G, has led us to these observations:

Received May 13, 1974.
Revised October 25, 1974.