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CONTINUOUS BOUNDARY BEHAVIOUR FOR FUNCTIONS DEFINED IN THE OPEN UNIT DISC

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This paper deals with cluster sets. While cluster sets can be considered in a more abstract setting, we shall limit ourselves to the study of functions f defined in the open unit disc D of the complex plane and taking their values on the Riemann sphere $\bar{C}$. For p a point of the unit circle C, we denote by $C(f, p)$ the cluster set of f at p, i.e., the set of all values $w \in \bar{C}$ for which there is a sequence $\{z_n\}$, $z_n \in D$, such that $z_n \to p$ and $f(z_n) \to w$. The point p is called a point of determination for f if $C(f, p)$ is a singleton. In Section 1 we characterize the set of points of determination of a function f defined in D. Namely, it is shown that the set of points of determination is a G_δ set, and conversely that given any G_δ set E on C, there exists a bounded holomorphic function f on D, whose set of points of determination is precisely E. We then consider the class of functions meromorphic in D which have the property that each point of the unit circle is a point of determination, or what is equivalent the class of functions continuous from $\bar{D}$ to $\bar{C}$ and meromorphic in D.

In Section 2 we fix f and consider $C(f, p)$ as a set-valued function of the variable p. From a theorem of M. K. Fort we conclude that $C(f, \cdot)$ is continuous on a residual G_δ set of the unit circle. Conversely, given any residual G_δ set E on C, there exists a bounded holomorphic function f such that E is precisely the set of points of continuity of $C(f, \cdot)$.

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