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ON THE TRANSITIVE DOMINATION PRINCIPLE FOR CONTINUOUS FUNCTION-KERNELS

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1. Introduction

One of the interesting problems in the potential theory for a function-kernel is to investigate the relation between the domination principle and the balayage principle.

N. Ninomiya first proved the equivalence of the above two principles for a positive symmetric continuous kernel using the Gauss-Ninomiya variation (cf. [6]).

For a positive non-symmetric lower semi-continuous kernel, M. Kishi established a new existence theorem and proved in [4] that a kernel G satisfies the domination principle if and only if G satisfies the balayage principle under the additional condition that G and its adjoint $\check{G}$ satisfy the continuity principle.

For a continuous function-kernel (in the extended sense), it is well known that a kernel G itself satisfies the continuity principle when G satisfies the domination principle. Does the adjoint $\check{G}$ satisfy the continuity principle? M. Itô and the author proved in [2] that $\check{G}$ also satisfies the continuity principle when G satisfies the domination principle. We used there a result obtained in [3] where M. Itô developed the theory of generalized kernels by means of elementary kernels and resolvents.

Let G and N be continuous function-kernels. In this paper we shall prove that G satisfies the relative domination principle with respect to N if and only if $\check{G}$ satisfies the transitive domination principle with respect to $\check{N}$. The analogous result is first obtained by M. Kishi under the additional condition that G and $\check{G}$ satisfy the continuity principle (cf. [4]).

Our result shall give a simple proof of the theorem obtained in [2] that G satisfies the domination principle if and only if $\check{G}$ satisfies the

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