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ON A PROBLEM OF BONAR CONCERNING FATOU POINTS FOR ANNULAR FUNCTIONS

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The purpose of this paper is to study the distribution of Fatou points of annular functions introduced by Bagemihl and Erdös [1]. Recall that a function $f(z)$, regular in the open unit disk $D: |z| < 1$, is referred to as an annular function if there exists a sequence $\{J_n\}$ of closed Jordan curves, converging out to the unit circle $C: |z| = 1$, such that the minimum modulus of $f(z)$ on J_n increases to infinity. If the J_n can be taken as circles concentric with C, $f(z)$ will be called strongly annular.

As a direct consequence of the definition, an annular function $f(z)$ can have at most one Fatou point, i.e., a point on C at which $f(z)$ has an angular limit, on any subarc of C which has no limit points of zeros of $f(z)$ and the Fatou value of $f(z)$, i.e., the angular limit, must be the point at infinity ∞. A natural question arises concerning to this observation: Does there really exist a Fatou point for every annular function? Bagemihl and Erdös [1] constructed an example of a strongly annular function which has no Fatou points. Many examples of annular functions are considered in Bonar [3], but it is not clear whether they have Fatou points or not. For these reasons Bonar [3] asks whether the value ∞ is a Fatou value for some annular function. The aim of this paper is to give an affirmative answer to this question of Bonar.

We shall recall first some elementary properties of annular functions. Every annular function $f(z)$ has infinitely many zeros and consequently their limit points form a non-empty closed subset, say $Z'(f)$, of C. We know that the set $Z'(f)$ coincides with the full circle C for almost every $f(z)$ of well known examples of annular functions. But as is shown in [2] and [5] there really exists an annular function $f(z)$ such that $Z'(f)$ does not coincide with C. For such $f(z)$, the complement of $Z'(f)$ on C consists of at most a countable number of disjoint open arcs. How-

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