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ON COMPLETENESS OF HOLOMORPHIC PRINCIPAL BUNDLES

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§ 0. Introduction

In this paper we shall investigate the structure of complex Lie groups from function theoretical points of view. A. Morimoto proved in [10] that every connected complex Lie group G has the smallest closed normal connected complex Lie subgroup G_e, such that the factor group G/G_e is Stein. On the other hand there hold the following two basic structure theorems (A_1) and (A_2) for a connected algebraic group G (cf. [12]). (A_1): G has the smallest normal algebraic subgroup N such that the factor group G/N is an affine algebraic group. Moreover N is a connected central subgroup. (A_2): G has the unique maximal connected affine algebraic subgroup L, where L is normal and the factor group G/L is an abelian variety.

It is well known that for algebraic groups "affine" and "linear" are equivalent, but for complex Lie groups "affine" i.e. "Stein" does not imply "linear" although the converse implication is true (cf. [9]). Nevertheless we may roughly say that Stein groups correspond to affine algebraic groups. In such a sense Morimoto's result can be considered as the analytic version of (A_1). Considering that complex tori correspond to abelian varieties, we can formulate the analytic version of (A_2), which is, however, not true in general. We shall prove, for reductive complex Lie groups, a structure theorem (Theorem 23) analogous to (A_2). In general every connected complex Lie group G admits a bundle structure over a complex torus, whose fibre is a Stein manifold and whose structure group is a Stein subgroup of G (Corollary 25). We are mainly concerned about the dimension of the complex torus which appears in the above fibreing of G. We shall find that it is independent of the group structure of G, but depends only on the underlying complex struc-

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