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# FREELY ACTING AUTOMORPHISMS OF ABELIAN $C^*$-ALGEBRAS

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## 1. Introduction.

Very recently, M. Choda, I. Kasahara and R. Nakamoto [3] extend the concept of free action of automorphisms for $C^*$-algebras and prove several theorems which are hither to known for von Neumann algebras. In the present note, we shall concern with freely acting automorphisms on abelian $C^*$-algebras. In § 2, several equivalent conditions for the free action are obtained. In § 3, we shall apply them to an automorphism which has a transversal group.

## 2. Equivalent conditions.

Let $A$ be a unital abelian $C^*$-algebra and $X$ be the character space of $A$, i.e. the compact space of all characters (multiplicative states) of $A$ equipped with the weak* topology.

Following after [2], [3] for an automorphism $\alpha$ on $A$, an element $a \in A$ is called a dependent element of $\alpha$ if

$$ax = x^\alpha a \tag{1}$$

is satisfied for every $x \in A$; if every dependent element of $\alpha$ is automatically 0, then we say that $\alpha$ is freely acting.

An automorphism $\alpha$ of $A$ naturally induces a homeomorphism of $X$ onto itself by

$$\chi^\alpha(x) = \chi(x^\alpha) \tag{2}$$

for every $\chi \in X$ and $x \in A$. Therefore, we shall consider $\alpha$ as an automorphism of $A$ and a homeomorphism of $X$ onto itself. For a set $U \subset X$ (resp. $K \subset A$) we shall denote $U^\alpha = \{\chi^\alpha ; \chi \in U\}$ (resp. $K^\alpha = \{x^\alpha ; x \in K\}$).

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