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ON A CLASS OF DEGENERATE ELLIPTIC EQUATIONS

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We shall prove in Chapter I the hypoellipticity[1] for a class of degenerate elliptic operators of higher order. Chapter II will be devoted to the consideration of the regularity at the boundary for the solutions of general boundary problems for the equations considered in Chapter I being restricted to the second order.

Chapter I. Hypoellipticity for a class of degenerate elliptic operators.

§ 1. Introduction.

In [5], Grušin has proved the hypoellipticity for a class of degenerate elliptic equations. Our aim in this chapter is to give a simple proof with some additional assumptions on the operators considered in [5].

First we state the main result obtained in [5]. Let $\boldsymbol{R}^N$ be N-dimensional Euclidean space regarded as a direct product of two Euclidean spaces $\boldsymbol{R}^k$ and $\boldsymbol{R}^n$ $(k + n = N)$. We consider a pair (ρ, σ) of N rational numbers $(\rho_1, \cdots, \rho_N)$, $(\sigma_1, \cdots, \sigma_N)$ such that $\rho_j \geqq 1$ and $\sigma_j \geqq 0$ $(1 \leqq j \leqq N)$ and that

$$\text{(a)} \qquad \rho_j = \sigma_j = 1 \qquad k + 1 \leqq j \leqq k + n = N$$

and for each $j, 1 \leqq j \leqq k$, one of the following conditions is satisfied:

$$\text{(b)} \qquad \rho_j > \sigma_j > 0 \,,$$

$$\text{(c)} \qquad \sigma_j = 0 \,.$$

Suppose (ρ, σ) is given. The following notations are convenient for the later discussions:

$$x = (x_1, \cdots, x_N) \in \boldsymbol{R}_N \,,$$

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[1] A differential operator P is said to be hypoelliptic, if any distribution u is infinitely differentiable in every open set where Pu is infinitely differentiable.