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Nagoya Math. J.
Vol. 55 (1974), 161–179

# ON WEAK CONCEPTS OF STABILITY

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## § 1. Introduction.

The manifold in this paper is assumed to be connected differentiable of class $C^\infty$. Let $\mathscr{D}^r(M)$ and $\mathfrak{X}^r(M)$ be the set of all diffeomorphisms and vector fields of class $C^r$ on a manifold $M$ with Whitney $C^r$ topology, respectively. In [2], the concept of weak stability is defined. The definition is equivalent to the following ((2.1) of this paper); $f \in \mathscr{D}^r(M)$ or $X \in \mathfrak{X}^r(M)$ is *weakly* (*allowably*) *stable* if and only if there is a neighborhood $U$ of $f$ or $X$ in $\mathscr{D}^r(M)$ or $\mathfrak{X}^r(M)$ such that for any (a suitable) $g$ or $Y \in U$ the set of all elements topologically equivalent to $g$ or $Y$ is dense in $U$, respectively. Here, $f, g \in \mathscr{D}^r(M)$ are said to be topologically equivalent if they are topologically conjugate and $X, Y \in \mathfrak{X}^r(M)$ are said to be topologically equivalent if there is a homeomorphism mapping any trajectory of $X$ onto a trajectory of $Y$ preserving the orientations of the trajectories. Similarly, *weak $\Omega$-stability* is defined for $f$ and $X$.

All weakly ($\Omega$-) stable systems compose an open set. The set of all systems which are weakly ($\Omega$-) stable but not structurally (or $\Omega$-) stable is also an open set. In [2] it is shown that in case of the non-wandering set being finite the weak stability of a diffeomorphism of a compact manifold implies the structural stability. The nondensity of weakly stable diffeomorphisms and weakly $\Omega$-stable diffeomorphisms are shown in [2].

One aim of this paper is to prove some results ((2.2), (3.2), and (4.2)) about allowable stability which are similar to some results mentioned in [2] with or without proof. Another aim is to prove the existence of a vector field on the 2-plane $\boldsymbol{R}^2$ which is weakly stablc but not structurally stable. This example is mentioned in [2] without proof.

## § 2. Weak stability and allowable stability.

Let $T$ be any topological space and $\sim$ be an equivalence relation