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SIEGEL DOMAINS OVER SELF-DUAL CONES AND THEIR AUTOMORPHISMS

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Introduction

The Lie algebra $\mathfrak{g}_h$ of all infinitesimal automorphisms of a Siegel domain in terms of polynomial vector fields was investigated by Kaup, Matsushima and Ochiai [6]. It was proved in [6] that $\mathfrak{g}_h$ is a graded Lie algebra; $\mathfrak{g}_h = \mathfrak{g}_{-1} + \mathfrak{g}_{-1/2} + \mathfrak{g}_0 + \mathfrak{g}_{1/2} + \mathfrak{g}_1$ and the Lie subalgebra $\mathfrak{g}_a$ of all infinitesimal affine automorphisms is given by the graded subalgebra; $\mathfrak{g}_a = \mathfrak{g}_{-1} + \mathfrak{g}_{-1/2} + \mathfrak{g}_0$. Nakajima [9] proved without the assumption of homogeneity that the non-affine parts $\mathfrak{g}_{1/2}$ and $\mathfrak{g}_1$ can be determined from the affine part $\mathfrak{g}_a$.

The main purpose of the present paper is to determine explicitly the Lie algebras $\mathfrak{g}_h$ for Siegel domains over self-dual cones. In § 2 we will prove that if the adjoint representation ρ of $\mathfrak{g}_0$ on $\mathfrak{g}_{-1}$ is irreducible, then $\mathfrak{g}_h$ is simple or $\mathfrak{g}_h = \mathfrak{g}_a$ (Theorem 2.1). Moreover using Nakajima's result we will give sufficient conditions of the vanishing of $\mathfrak{g}_{1/2}$ (Proposition 2.3 and Corollary 2.7) and a method of calculating $\mathfrak{g}_{1/2}$ and $\mathfrak{g}_1$ (Propositions 2.6 and 2.8). Using the results in § 2, we determine in § 3 (Theorems 3.3–3.6) infinitesimal automorphisms of most of the homogeneous Siegel domains over self-dual cones (other than circular cones) which were constructed by Pjateckii-Sapiro [10].

The *circular cone* $C(n)$ of dimension n $(n \geq 3)$ is defined to be the set $\{{}^t(x_1, x_2, \cdots, x_n) \in \boldsymbol{R}^n; x_1 > 0, x_1x_2 - x_3^2 - \cdots - x_n^2 > 0\}$. Pjateckii-Sapiro [10] found all the homogeneous Siegel domains over circular cones which are constructed by using the representation theory of Clifford algebras. But it was shown by Kaneyuki and Tsuji [5] that there exists a homogeneous Siegel domain over a circular cone which does not appear in Pjateckii-Sapiro's construction. In view of this fact the purpose in § 4 is to give a method of constructing all homogeneous Siegel domains over

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