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ON THE ZEROS OF A CONFORMAL VECTOR FIELD

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1. Introduction

In [1] S. Kobayashi showed that the connected components of the set of zeros of a Killing vector field on a Riemannian manifold (M^n, g) are totally geodesic submanifolds of (M^n, g) of even codimension including the case of isolated singular points. The purpose of this short note is to give a simple proof of the corresponding result for conformal vector fields on compact Riemannian manifolds. In particular we prove the following

THEOREM. *Let (M^n, g) be a compact Riemannian manifold of dimension $n \geq 2$. Let F be the set of zeros of a conformal vector field ξ and let $F = \bigcup V_i$ where the V_i's are the connected components of F. Then each V_i is either an umbilical submanifold of (M^n, g) of even codimension including the case of isolated singular points or an isolated singular point of a conformal non-Killing vector field on a Euclidean sphere.*

The idea of our proof is to reduce the problem to Kobayashi's case by a simple application of a theorem of M. Obata characterizing a sphere to conformality. In Section 2 we discuss Obata's result and then prove our theorem in Section 3.

2. Preliminaries

A Riemannian metric $\bar{g}$ is said to be *conformal* to g if there exists a smooth function ρ on M^n such that $\bar{g} = e^{2\rho}g$. Let $f: M^n \to M^n$ be a diffeomorphism of M^n onto itself; we say f is a *conformal diffeomorphism* if f^*g is conformal to g.

Let $C(M^n, g)$ denote the Lie group of all conformal diffeomorphisms of (M^n, g) and $C_0(M^n, g)$ the connected component of the identity. A

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