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AN APPLICATION OF THE MORSE THEORY TO FOLIATED MANIFOLDS

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In [5], R. Thom has started the study of the foliated structures by using the Morse theory. Recently K. Yamato [7] has studied the topological properties of leaves of a codimension one foliated manifold by investigating the "critical points" of variation equation of the given one-form.

In this note, using their methods we shall show that a codimension k foliation on a closed manifold is a "bundle foliation" under certain conditions (Theorem I), and give some topological properties of those leaves (Theorem II, III). By using Theorem I, we shall show the Stability Theorem of Reeb [3]. Furthermore, we shall show that bundle foliations satisfying some conditions, are stable under a small perturbation (Theorem IV). All manifolds, foliations and mappings considered here, are smooth (i.e., differentiable of class C^∞).

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§ 1. Definitions and statement of the results

Let V^n be a closed n-manifold, $\mathscr{F}^k$ $(0 < k < n)$ a codimension k foliation on V^n and $\{U_i, i = 1, 2, \cdots, \ell_0\}$ a distinguished neighborhood covering of V^n. The local coordinate of a distinguished neighborhood is $(u_1, \cdots, u_k, x_1, \cdots, x_{n-k})$ such that each plate is defined by $u_i =$ constant for $1 \leqq i \leqq k$. At first we take a smooth function f on V^n, and for each distinguished neighborhood U_i, we define a mapping F_i of U_i into $\boldsymbol{R}^{k+1}$ by

$$F_i(u_1, \cdots, u_k, x_1, \cdots, x_{n-k}) = (u_1, \cdots, u_k, f(u_1, \cdots, u_k, x_1, \cdots, x_{n-k})) .$$

Then, we define a subset $\Gamma(f)$ of V^n as follows: $\Gamma(f) \cap U_i = \{(u_1, \cdots, u_k,$

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