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ON THE MODULE STRUCTURE OF THE RING OF ALL INTEGERS OF A $\mathfrak{p}$-ADIC NUMBER FIELD

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Let k be a $\mathfrak{p}$-adic number field and $\mathfrak{o}$ be the ring of all integers of k. Let K/k be a cyclic ramified extension of prime degree p with Galois group G. Then the ring $\mathfrak{O}$ of all integers of K is $\mathfrak{o}[G]$-module. The purpose of this paper is to give a necessary and sufficient condition for $\mathfrak{o}[G]$-*module* $\mathfrak{O}$ *to be indecomposable.*

In §§ 1–2, we shall prepare some lemmas. In § 3, we shall obtain the necessary and sufficient condition (Theorem 1).

1. In this section, we shall construct an arithmetical sequence of rational integers and study its properties. We begin with defining sequences $a_1^i, a_2^i, \cdots, a_{p-1}^i$ for $1 \leqq i \leqq p-1$. Sequences a_j^i are defined inductively by:

$$a_1^1 = 1,\ a_2^1 = 2, \cdots, a_{p-1}^1 = p-1$$
$$a_1^2 = 0,\ a_2^2 = a_1^1,\ a_3^2 = a_1^1 + a_2^1, \cdots, a_{p-1}^2 = a_1^1 + a_2^1 + \cdots + a_{p-2}^1, \cdots,$$
$$a_1^i = 0, \cdots, a_{i-1}^i = 0,\ a_i^i = a_{i-1}^{i-1},\ a_{i+1}^i = a_{i-1}^{i-1} + a_i^{i-1}, \cdots,$$
$$a_{p-1}^i = a_{i-1}^{i-1} + a_i^{i-1} + \cdots + a_{p-2}^{i-1}, \cdots .$$

We evaluate a_j^i.

LEMMA 1.

$$a_j^i = \frac{\{j-(i-1)\}\{j-(i-2)\}\cdots j}{i!} \qquad \textit{for } 1 \leqq i \leqq j \leqq p-1 .$$

Proof. We use induction on i. The result is trivial for $i=1$. Let $i>1$, and suppose the result holds for $a_j^{i'}$ where $1 \leqq i' \leqq i-1$ and $i' \leqq j \leqq p-1$. Then we have

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